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Buying from the Fringe (too)

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Abstract

We analyze how to divide the requirements of a (public) firm into lots, when potential suppliers suffer from heterogeneous diseconomies of scale. The optimal design leads to all firms, included the disadvantaged competitors, the fringe, being active, despite the concomitant cost of increasing supplier profit. Setting large lots that only large firms can produce competitively is necessary; but also setting small lots that the fringe firms can competitively bid for, reduces procurement cost. If, in addition, some medium-sized lots are set aside for the fringe – as allowed by the US regulations, but not by the EU ones – procurement cost is further reduced.

JEL: L13, L51, D47, K23.

Keywords: Procurement, sequential auctions, block sourcing, regulation, affirmative action.

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1 Introduction

In this paper we discuss how to reduce the procurement costs via sequential auctions in a setting where both large – and thus more efficient at producing large quantities – and small suppliers – that we refer to as the *fringe* – are present. We consider the situation where the requirements are a fixed amount of a divisible commodity, there are negligible fixed costs and (short-run) marginal costs are increasing, so that it is efficient to have all the firms, including the small ones, producing. We pay particular attention to how existing regulation affects the effectiveness of these mechanisms.

The historical approach in this scenario has been to auction off the contract for supplying the entire requirements to a single winner, or at best setting a few large contracts, for which the fringe firms could not be competitive. This method is successful in minimizing the winning suppliers' profits; however it leads to very inefficient production, by excluding the small (and some of the large) firms – who, consequently, have lower marginal cost than the producing large firms. The concomitant under-representation of small and medium sized enterprises (SME) in public procurement is particularly salient in Europe as Figure 1 illustrates.

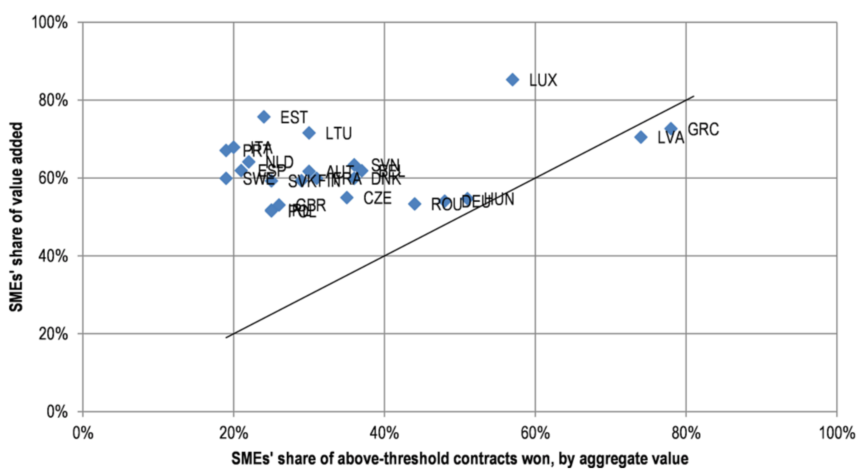

Figure 1. Underrepresentation of small and medium size european firms in public procurement. Source, OCDE (2018).

To redress this situation, the EU Public Sector Directive 2014/24/EU promotes SMEs participation in public procurement. For example, reducing the administrative burdens and costs of participation (fixed costs) is seen as important. It also aims at increasing SME participation in public procurement, by the division of public contracts into many small lots, so that the SME can be competitive bidding for them. We model the outcome of such a situation as a competitive market, where all the suppliers produce and their equalized marginal costs determine the price. It is immediate that this mechanism leads to efficient production; however, due to the increasing marginal costs, it gives positive profits to the suppliers. Indeed, there is a common presumption that rules favoring SMEs increase procurement cost for public agencies (at least in the short run).[1]

To the contrary – building on the insights from Bru et al. (2023)[2] – below we show that combining the two mechanisms, that is, organizing an auction for the large firms followed by auctioning off some sufficiently small lots, reduces total procurement cost (compared to either mechanism on its own). In other words, when implemented judiciously, the EU directive is not a burden on public administrations. There are two reasons for this result: First, SMEs can be quite efficient at producing small quantities; it is when asked to produce high quantities that they face stepping costs compared to large firms. Second, incorporating SMEs to the procurement process provides competition for the large bidders ending up without a large lot, and thus indirectly forces all large firms to reduce their bids for the large lots.

At the same time as the EU wants to promote SMEs participation in public procurement, European economic law adheres to the strict principles of non-discrimination. In the US, instead, there is since 1978 (Public Law 95-507 that amended the 1958 Small Business Act), a programme of affirmative action that includes preferential treatment of SMEs in public procurement. In particular, at the Federal level of government, each fiscal

[1]Policies that promote the participation of small firms in public procurement can improve their long-term performance. di Giovanni et al. (2022) show that they help them to grow and overcome financial constraints. Gil and Marion (2013) find that prior subcontracting interactions reduce bids for California highway procurement contracts; what suggests that the experience acquired as subcontractors improve their future performance (see Rosa, 2020, for a theoretical analysis of this possibility).

[2]In that paper, all potential suppliers are equal, no fringe is contemplated.

year the Small Business Administration negotiates with the Federal agencies "set-asides" or "quotas", i.e., a minimum level of procurement that must go to SMEs.[3]

We show that set-asides for SMEs, along the lines of the discriminatory measures contemplated by the US administration, can further reduce procurement costs. Thus, we provide an additional argument against the presumption that procurement cost minimization and the inclusion of SMEs are incompatible. We implement the cost-reducing set-aside by setting some medium-sized lots for SMEs only, in addition to the large lots and the small lots that are similar to the "European" system above. That is, our mechanism with discrimination starts with a large-lot auction (aimed at large firms), which is followed by a medium-lot auction restricted to SMEs and is finished by a competitive market. The resulting reduction in procurement costs is the sum of two effects. First, productive efficiency is increased: firms produce closer to their competitive quantities. Second, the profits of (all) the firms are reduced. The latter result is perhaps surprising: a mechanism that on the face of it favors SMEs by protecting them from the direct competition of large firms, actually hurts their profitability.[4] Nonetheless, we also show that if the public agency also worries about SMEs profits, it can readjust the quantity set-aside and still end up with lower procurement costs while increasing SMEs profits compared with a situation in which set-asides are banned.

In the next section we review the closely related literature. In Section 3 we present our set-up, and in Section 4 we provide the benchmark results, for the auction-only and the competitive market. Section 5 has the detailed analyses of the proposed sequential auctions. We close the paper with some concluding remarks. All proofs not included in the main text are collected in an Appendix.

[3] See https://www.sba.gov/federal-contracting/contracting-guide/types-contracts

[4] This effect is somewhat compensated by the fact that, for most parameter configurations, SMEs produce more on the whole with the set-aside auction included.

2 Related literature

The existing empirical literature has ambiguous results on the impact of discriminatory policies in favor of SMEs on procurement costs:[5] in Nakabayashi (2013) and Mummalaneni (2022) they reduce it, while Marion (2007) and Athey et al. (2013) obtain the opposite result.

In Nakabayashi (2013) a counterfactual analysis demonstrates that – in Japan – approximately 40 percent of SMEs would exit the procurement market if set-asides were removed, and that the resulting lack of competition would increase government procurement costs. Similarly, Mummalaneni (2022) argues that public agencies tend to limit their purchases when discounts for SMEs are present, fearing that their expenditure would increase. To the contrary, he shows that public agencies in Virginia would reduce their procurement expenditures by roughly 12 percent if they used a stronger affirmative action policy, as this would intensify competition and force large, low-cost vendors to significantly reduce their prices.

On the other hand, Athey et al. (2013), who analyze US Forest Service timber auctions, estimate that set-asides substantially reduce efficiency and revenue. While they increase small business participation, an alternative policy of subsidizing small bidders would increase revenue and small bidder profit, with little efficiency cost. In this vein, Marion (2007) establishes that in California the 5 percent subsidy that accrues to small businesses – in auctions for road construction projects using only state funds – increases the procurement costs by 3.8 percent compared to projects using federal aid, where there is no such bid preference program.

All in all, this literature suggests that helping SMEs to participate in the procurement process may reduce overall costs, but that the correct design of the procurement process (possibly adapted to the particulars of the product procured) is crucial to obtain these

[5] The participation of SMEs in public procurement can also be indirect, as when there are subcontracting regulations that force a prime contractor to set aside the production of a share of the contract that it wins for subcontractors designated by the Administration as disadvantaged. Other measures to promote SMEs participation in public procurement are subsidizing the bids of SMEs, and mandatory rules for lot winners to subcontract part of the production to SMEs.

efficiencies.

Previous theoretical literature also highlights the importance of preferential treatment and affirmative action. Both the mechanism design (see Myerson, 1981, and McAfee and McMillan, 1987) and the contest literature (Dahm and Esteve-González, 2018, Alcalde and Dahm, 2020, and Chowdhury et al., 2022) show that handicapping efficient bidders can reduce procurement costs through their effect on overall competition. Unlike us, all these papers consider constant marginal costs of production, and the role of favoring weak agents in reducing procurement costs is similar in all of them: cost efficiency dictates sole sourcing by the more efficient supplier when marginal costs are constant; but then this supplier can set a winning bid only slightly below the marginal cost of its rivals, possibly quite above its own marginal cost. If part of the production is set aside for less efficient suppliers in an amount that depends on all bids, then the efficient supplier has incentives to lower its bid to increase its production share. Perry and Sákovics (2003) and Jehiel and Lamy (2020) show that, in addition, set-asides for inefficient suppliers can further reduce procurement costs if they lead to additional entry of potential suppliers.

3 The set-up

Consider a buyer who wishes to procure X units of an infinitely divisible homogeneous good at the lowest price. There are two types of potential suppliers he can turn to. There is a group of $n_k > 1$ identical "large" suppliers with a strictly increasing, strictly convex and thrice differentiable production cost function $C(x) \equiv kc\left(\frac{x}{k}\right)$, with $c(0) = c'(0) = 0$ and $k > 1$, and a fringe of $n_1 > 1$ "small" suppliers with cost function $c(x)$.[6] We will focus our analysis on the situation where the difference in size/efficiency between large and small firms is significant ($k >> 1$), so that the fringe is indeed a fringe. An interpretation is that each plant of production has the same efficiency, with n_1 suppliers that own only

[6] To ensure that second-order conditions are globally satisfied (what greatly simplifies the analysis), we make the standard assumption that $c'''(.)$ is not too negative (the level of convexity does not decrease too fast in quantity): $c''(x) > -xc'''(x)$. Note that the most natural situation is that $c'''(.) > 0$, so the assumption is rather innocuous.

one plant, and n_k suppliers that own several, k, plants.[7] Note that from a production efficiency point of view the optimal solution would be to have each *plant* produce the same amount.

The cost structure described above combines diseconomies of scale "in the small" with economies of scale "in the large": within a plant/small firm we assume that there is a soft capacity constraint – given the capital installed, ramping up production is increasingly costly – but with additional capital/plants the same quantity can be produced at lower cost.

4 The benchmarks

As we have discussed above, in practice there are two popular ways to organize procurement. Either an auction is held for large lots designed for the efficient firms, practically ignoring the existence of small firms, or the requirement is divided into many small contracts, where all the firms can end up with several of them. We will use these mechanisms as benchmarks against which to compare our proposed procedure(s).

4.1 A large-lot auction

Suppose that an auction is organized for the large firms (only). It is straightforward to see that the optimal auction – ignoring the fringe – is of $n_k - 1$ lots of equal size, $\frac{X}{n_k-1}$.[8] As the "loser" earns zero profit, the buyer can extract all the surplus from the $n_k - 1$ firms that produce, and thus the equilibrium bidding leads to a procurement cost of

$$PC_A = (n_k - 1)\, C\left(\frac{X}{n_k - 1}\right) = k\,(n_k - 1)\, c\left(\frac{X}{k\,(n_k - 1)}\right),$$

what simplifies to

$$\frac{X^2}{2k\,(n_k - 1)}, \tag{1}$$

when[9] $c(x) \equiv 0.5x^2$.

[7] However, we do not restrict k to be an integer.

[8] Having n_k lots would eliminate the competitive pressure. Symmetry is efficient.

[9] We will use this cost function to illustrate our results throughout the paper.

It is immediate to see that, due to their cost handicap, small firms are not competitive in such an auction. Consequently, this mechanism allows for two productive inefficiencies: Neither the loser nor the small firms are producing, despite having much lower marginal costs of production – zero – than the winners of the auction, $c'\left(\frac{X}{k(n_k-1)}\right) >> 0$, who are the (only) ones producing.[10]

4.2 A "competitive market" for all the firms

A common alternative to the large-lot auction is to divide the requirements into small lots (and to allow for a firm to win multiple ones).[11] When these lots are sufficiently small, so that the marginal costs of all firms can be equalized while supplying X in the aggregate, the competitive outcome is the unique equilibrium.[12] It is immediate that if the buyer were able to set the price (only), he would choose the competitive price, as it is the lowest price for which he could satisfy his requirements. That is, price setting by the buyer would also lead to the competitive outcome, just as the auction of small units.

Let us denote by Q and q the quantities produced in equilibrium by a large and a small firm, respectively. By the definition of competitive equilibrium, marginal costs equalize and equal the price

$$p = c'(q) = c'\left(\frac{Q}{k}\right) = C'(Q), \tag{2}$$

implying that $kq = Q$ – that is, all the plants produce the same amount; and the market

[10] In principle, the buyer could set n_k large lots and allow the small firms to bid for them. In this case, all the large firms would produce but the price would be determined by the reservation price of the small suppliers. For k sufficiently small – that is, when the size difference between large and small firms is not significant – this might lower the cost for the buyer. We do not consider this scenario, as we wish to focus on the case where the large and small firms are qualitatively different. See Example 1 below to get an idea of k required.

[11] Note that this is not the (strong) classical assumption of not being able to affect the price, as it is the size of lots, not the suppliers' capacity, that is infinitesimal.

[12] See Burguet and Sákovics (2017) for an analysis with infinitesimal units. The idea is simple: if the price was higher than the marginal cost of some firms, they would be willing to underbid it on some units. If it was lower, they would increase their bids on some of the units so that their production will decrease to where price equals their marginal cost.

clears

$$n_k Q + n_1 q = X. \tag{3}$$

We then have $(kn_k + n_1)\, q = X$, so that $PC = pX = c'\left(\frac{X}{kn_k+n_1}\right) X$. Let us state this as a proposition.[13]

Proposition 1 *In the competitive market the large and small firms produce $Q_M = kq_M$ and q_M units, respectively, where $q_M \equiv \frac{X}{kn_k+n_1}$. The total procurement cost is*

$$PC_M = c'(q_M)\, X,$$

while supplier profits are

$$\pi_1 = \frac{\pi_k}{k} = c'(q_M)\, q_M - c(q_M).$$

The competitive outcome is – of course – efficient; its only "weakness" is that it allows "too much" profit for the suppliers: due to the convex cost function, even if price equals marginal cost, the suppliers make a profit on the inframarginal units. As each plant produces the same amount – and given the common price per unit – they also generate the same profit, and thus a firm with k plants earns k times the profit of a single-plant firm.

Corollary 1 *When $c(x) \equiv 0.5x^2$, procurement cost in a single market is*

$$PC = \frac{X^2}{kn_k + n_1}, \tag{4}$$

while supplier profits are

$$\pi_1 = \frac{\pi_k}{k} = 0.5\left(\frac{X}{kn_k + n_1}\right)^2.$$

Comparing (1) with (4), it is straightforward to see that this procurement policy may or may not decrease total costs relative to the large-lot auction, depending on the (relative) size of the fringe and its cost handicap. More precisely, the market is better for the buyer if the size of the fringe is greater than the size of the large firms excluding two of them.

[13] While we are not considering it an alternative, as we wish to avoid discrimination in a market, it is straightforward to see that separate markets for large and small firms would lead to the same outcome, as with the optimal split of the demand between the two markets, marginal costs would equalize across markets (and equal the common price).

Corollary 2 *When $c(x) \equiv 0.5x^2$, the market solution reduces procurement costs relative to an auction for the large firms only if and only if $n_1 > k(n_k - 2)$.*

The two mechanisms discussed can be seen as the two (opposite) corner solutions: with the large lots the buyer can ensure that he extracts all the surplus, while incurring in inefficient production. Instead, with the market solution production is efficient but supplier profits "soar". It should come as no surprise that there is a superior solution where the trade-off between efficiency and supplier profits is optimized. This would not consist in setting medium sized lots though, but in combining large (auction) and small lots (competitive market)

5 Combining lot auctions and the market

In order to derive a mechanism that leads to a lower procurement cost than either of the simple procedures detailed above, we now consider a combination of the two. In principle, buying at different prices could be self-defeating: witness the Coase conjecture, where a monopolist cannibalizes her own sales today by offering to sell (cheaper) later as well (see Coase, 1972; Stokey, 1979). In our case, however, we are not considering simply dynamic pricing but mixing an auction with a price/market. The Coasian idea continues to hold when we put the price first: setting a price before the auction is not a good idea as no large firm would wish to handicap themselves – by raising their marginal cost – before the auction. On the other hand, as first shown in Bru et al. (2023) in a model without the fringe, the alternative sequence does allow for an optimal compromise between rent extraction and efficiency, as the loser of the auction is willing to engage with the market. In equilibrium all firms produce, while the sizes of the lots can be used to limit their profits. Since in equilibrium all suppliers with the same cost function earn the same in expectation, and since (at least) one of each supplier type participate in the market, decreasing the size of the residual demand – by increasing the lot size – controls the aggregate profit of all the suppliers.

5.1 A large-lot auction followed by a market

Consider that following a lot auction, for the residual demand there is a competitive market for the suppliers who have not won a lot in the auction (that is, one large and n_1 small firms). We now derive the optimal lot sizes and the resulting procurement cost, and show that the latter is always lower than the one resulting from either of our benchmarks.

Before proceeding, we wish to elaborate on an important point: In principle, the same arguments in favor of holding an auction before the market would support the idea of organizing a second auction – with smaller lots, so that the small firms are interested in bidding – still before the market. When k is close to 1, this would definitely work: think of the case of $k = 1$, where it is optimal to set $n_k + n_1 - 1$ equal lots, what is equivalent to two auctions that in sum have the same number of lots. Note that the efficient loser would always win a lot in the second auction. Moreover, when k is sufficiently large this efficient firm would prefer to participate in the market as well, destroying the profitability of adding a second auction.[14] In fact, at the point of indifference of the loser, the procurement cost would already be higher than with a single auction. To see this, note that in that case the loser would produce the same as if it participated only in the market, but would be paid more for it, as the lot price would be determined by the inefficient firms' bids. To simplify the analysis, we assume that k is indeed high enough, so that a second auction is not profitable. This is consistent with our focus on situations where the small firms are indeed a fringe. At the end of this subsection we provide an example for the level of k required.[15]

Proposition 2 *When k is sufficiently large, it is optimal to set a single auction, with $n_k - 1$ equal lots,*[16] *followed by a competitive market for the residual demand. The total amount auctioned, Z^*_{EU} satisfies*

$$c'\left(\frac{Z^*_{EU}}{k(n_k - 1)}\right) - \frac{kn_k + n_1}{k + n_1} c''\left(q^*_{EU}\right) q^*_{EU} - c'\left(q^*_{EU}\right) = 0, \tag{5}$$

[14] If a lot winner also supplies a positive amount in the market then this is equivalent to demanding the lot units through the market. See Lemma 2 in Bru et al. (2023).

[15] For a general cost function there is no explicit solution.

[16] Just as in the case of the large-lot auction, we do not consider the possibility to set n_k large lots. For k sufficiently large they would never be optimal. See Example 1.

where $q^*_{EU} = \frac{X - Z^*_{EU}}{k+n_1}$ *is the supply of the small firms. The resulting* Z^*_{EU} *and* q^*_{EU} *are positive and the procurement cost is*

$$PC\left(Z^*_{EU}\right) = k(n_k - 1)\left(c\left(\frac{Z^*_{EU}}{k(n_k-1)}\right) - c(q^*_{EU})\right) + c'(q^*_{EU})q^*_{EU}\left(kn_k + n_1\right). \tag{6}$$

When $c(x) \equiv 0.5x^2$, this simplifies to

$$Z^*_{EU} = X\frac{k\left(n_k - 1\right)\left(kn_k + 2n_1 + k\right)}{\left(kn_k + n_1\right)^2} \text{ and } q^*_{EU} = X\frac{k + n_1}{\left(kn_k + n_1\right)^2}$$

$$PC_{EU} = X^2\frac{kn_k + 2n_1 + k}{2\left(kn_k + n_1\right)^2}. \tag{7}$$

For a price reduction to occur due to the auction, the crucial detail is that the lot winner(s) should not wish to participate in the market, otherwise it is as if they only participated in the market. Thus, the optimal lot size is larger than efficient (the amount (large) firms would produce in the one-shot competitive market). Naturally, this implies that the firms that sell through the market, produce less than the efficient amount.

Corollary 3 *The optimal lot size exceeds the large firms' production in the market-only mechanism:*

$$kq_M < \frac{Z^*_{EU}}{n_k - 1}.$$

Proof. If we substitute the production in the competitive market of a large firm kq_M for $\frac{Z^*_{EU}}{n_k-1}$ in the first-order condition (5), the first derivatives cancel out, so we obtain a negative value: increasing the lot size beyond kq_M decreases the cost of procurement. ■

The construction of the optimal mechanism allows for $Z^* = 0$ and for $q^* = 0$, corresponding to no auction or no market, but they still turn out to be positive. Thus, Proposition 2 implies (even in the absence of explicit formulas for PC) that procurement costs are lower than either in the lot auction or in the market:

Corollary 4 *The procurement cost with an (optimally designed) auction followed by the market is lower than either the competitive market or the large-lot auction only.*

The following numerical example illustrates the condition on the minimal size difference to avoid setting a second auction being optimal:

Example 1 *Let us consider two inefficient suppliers with cost function* $c(x) \equiv \frac{x^2}{2}$ *and two efficient suppliers with cost function* $kc\left(\frac{x}{k}\right) \equiv \frac{x^2}{2k}$. *We normalize total quantity procured as* $X = 1$. *Consider now two lots, one of size* Z *and a second one smaller,* z. *The inefficient suppliers bid for the smaller lot making the equilibrium conjecture that the rival in the market is the other inefficient firm, and thus they would sell* $q = \frac{1-Z-z}{2}$ *in the market at price* $p = c'(q)$. *This leads to a bid of* $b = c(z) + c'(q)q - c(q) = 0.5\left(z^2 + q^2\right)$. *The equilibrium bids of the efficient suppliers are* $B_z = b$ *for the small lot (by the loser of the first auction, who just needs to match the small firms' bid) and* $B_Z = kc\left(\frac{Z}{k}\right) + b - kc\left(\frac{z}{k}\right) = \frac{Z^2-z^2}{2k} + b$ *for the larger one, where we have set the expected profits of the two large suppliers equal. Procurement costs are therefore*

$$PC(Z, z; k) = B_Z + B_z + q(1 - Z - z) = \frac{1}{2k}Z^2 + \frac{2k-1}{2k}z^2 + \frac{3}{4}(1 - Z - z)^2$$

The optimal sizes of the larger and smaller lot satisfy the first-order conditions $\frac{Z}{k} = \frac{3(1-Z-z)}{2}$ *and* $\frac{2k-1}{k}z = \frac{3(1-Z-z)}{2}$. *The resulting optimal lots are of size*

$$\{Z, z\} = \left\{\frac{3k(2k-1)}{6k^2+4k-2}, \frac{3k}{6k^2+4k-2}\right\},$$

and procurement costs are $PC(k) = \frac{3k(2k-1)}{4(3k^2+2k-1)}$. *Comparing these procurement costs with (7) – setting* $n_k = n_1 = 2$ *– it is straightforward to see that the two-auction solution is only better for* $k < 1.457$.

The above example also illustrates that setting as many (symmetric) lots as there are large suppliers generates larger procurement costs than just setting a lot for all except one large supplier.

5.2 A superior solution via a set aside

In the previous subsection we argued that one of the constraints on the profitable use of a second auction was that the losing large firm will always beat the small firms in "their" auction. We will rule it out now by considering that the second auction is "set aside" for the small firms. Of course, whether this is possible depends on the legislation: while in the US this is clearly legal, in the European Union it may be difficult to argue.

Proposition 3 *If one auction is restricted to small firms only, then the optimal auctions have $n_k - 1$ and $n_1 - 1$ equal lots each, and the total amounts auctioned, Z and Y, satisfy*

$$\frac{kn_k + n_1}{k+1} c''(q)\, q + c'(q) = c'\left(\frac{Z}{k(n_k - 1)}\right) = c'\left(\frac{Y}{n_1 - 1}\right), \tag{8}$$

where $q = \frac{X-Z-Y}{k+1}$. The resulting Z^*_{US}, Y^*_{US} and q^*_{US} are all positive, satisfy $q^*_{US} < q_M$ and $kq_M < \frac{Z^*_{US}}{k(n_k-1)} = \frac{Y^*_{US}}{n_1-1}$ and the procurement cost is

$$PC\left(Z^*_{US}, Y^*_{US}\right) = \left(k(n_k - 1) + n_1 - 1\right)\left(c\left(\frac{Y^*_{US}}{n_1 - 1}\right) - c\left(q^*_{US}\right)\right) + c'(q^*_{US})q^*_{US}\left(kn_k + n_1\right). \tag{9}$$

Observe that the per-plant production of lot winners is equalized across the two auctions, in line with (constrained) efficiency. When $c(x) \equiv 0.5x^2$ the proposition simplifies to

$$\frac{Z^*_{US}}{k(n_k - 1)} = \frac{Y^*_{US}}{n_1 - 1} = \frac{k(n_k + 1) + n_1 + 1}{(kn_k + n_1)^2} X,$$

$$X - Z^*_{US} - Y^*_{US} = \left(\frac{k+1}{kn_k + n_1}\right)^2 X$$

and

$$PC_{US} = \frac{k(n_k + 1) + (n_1 + 1)}{2(kn_k + n_1)^2} X^2. \tag{10}$$

By the construction of the optimal mechanism, Proposition 3 implies that procurement costs are minimized across all possible combinations of auctions and market (even in the absence of explicit formulas). Hence,

Corollary 5 *When setting aside is allowed, the two-auctions-plus-market solution is superior to any mechanism that consists of a subset of these three ingredients.*

5.3 Affirmative action

The mechanism identified in the previous subsection gives the impression that affirmative action and cost minimization go hand in hand. However, as we show below, the lower procurement cost of affirmative action is actually the result of two effects: more efficient production and lower supplier profits. Therefore, excluding the "ogre" from the small-lot auction does not benefit the small firms.

Corollary 6 *The small firms make a lower profit in the US system than in the EU system, and they make a lower profit in the EU system than in the market only mechanism.*

The key insight driving this result is that the US system is more efficient. In fact, the gain in efficiency is sufficient to allow for a handicap system, where the small firms are guaranteed their EU profits. From what we have seen it is clear that the modified system must involve a decrease in the size of lots, as that would increase the size of the residual market and thus increase supplier profits ("unfortunately" of both types of suppliers). This adjustment would also bring production closer to efficiency thereby increasing the size of the pie.

Corollary 7 *It is possible to lower lot sizes in the US system so that the small (and the large) firms make the same profit as under the European system, but procurement costs are still lower.*

The above results can be nicely illustrated by Figure 2, where on the horizontal axis we measure the quantity a small firm produces in the market, q, and then calculate the resulting procurement cost assuming the lots are sized optimally given q. The two extremes of the top curve correspond to our benchmarks, and the lowest point of each curve depicts the optimal solution in the EU and US systems. As we can see, the US procurement cost is below the EU one for all $q < q_M$. We can also see that setting $q = q^*_{EU}$ and therefore allowing the suppliers the same profits as in the EU system, the US system leads to lower procurement cost. In fact, we can increase the supplier profits further (until the horizontal line cuts the lower curve) and the US procurement costs will continue be lower than with the optimal EU lots policy.

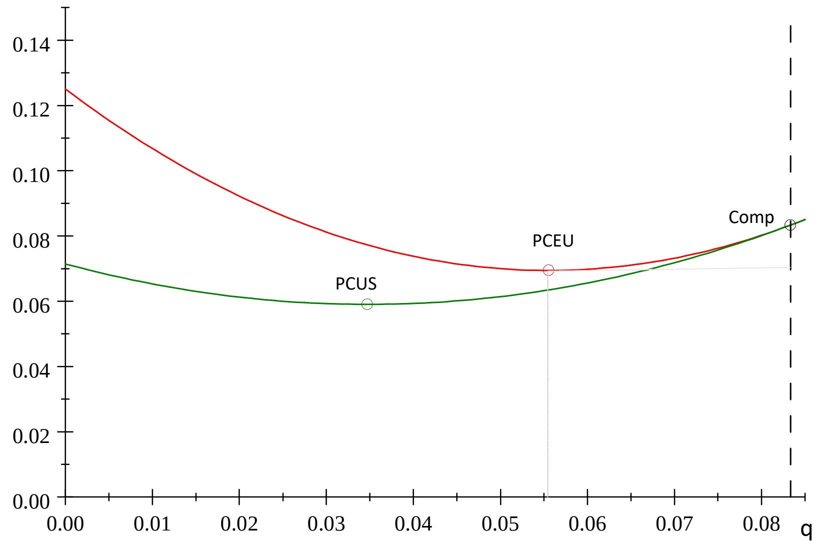

Figure 2. Procurement costs as a function of q when $c(q) = q^2/2$, $n_k = 2, n_1 = 4$, $k = 4$ and $X = 1$.

In contrast to the ranking of profits, the proportion of the requirements purchased from the fringe is likely to be higher in the US system. We state and prove this result only for our quadratic cost function.

Corollary 8 *When $c(x) \equiv 0.5x^2$, the small firms produce less in the European system than in either the market only mechanism or in the US system. Furthermore, if and only if there are more small than large firms, $n_1 > n_k$, their production with the US system exceeds that of the market only mechanism: $0 < n_1 q^*_{EU} < n_1 q_M < q^*_{US} + Y^*_{US}$.*

Since $n_1 > n_k$ is likely to be satisfied in applications, we can state that discriminating "in favor" of small firms indeed increases the proportion of the buyer's requirements that are procured from the small firms – and this may have some positive effects, say on their employment.

6 Concluding remarks

We have shown that the sequential deployment of simple auctions can lead to a favorable – to the buyer – resolution of the trade-off between efficient production and supplier profit minimization. Though we restricted attention to the simple case of two types of suppliers, our insight that a sequence of lot auctions in decreasing sequence lowers procurement costs is general. We have also provided arguments in favor of inclusive policies for SMEs. While we framed our discussion in terms of public procurement – as the regulations discussed apply to them –, the analysis of our mechanisms equally applies if the buyer is a private enterprise. Note that the institution of signing large contracts first and organizing a spot market later – usually driven by sequential resolution of uncertainty, what is not present in our model – is common and it fits our prescription as well.

Our analysis has not addressed other possible measures that could fit into the European law and favor the participation of SMEs in public procurement. In particular, the European Directive 2014/24/EU contemplates that participation limits ("a limit in the number of lots an economic operator may tender") and award limits ("to limit the number of lots that may be awarded to any one tenderer") be allowed in European public procurement (for a discussion of these tools, see Albano, 2019). Also, we did not address the possible role of joint bidding and mergers by SMEs. In this respect, it can be shown in our set-up that by replacing k small suppliers with a large supplier, the procurement costs are not affected under the US system whereas they decrease in the European system.[17] This fact informs us about the effects that these agreements between SMEs could generate on public procurement.

[17] For an illustration, see how this merger would change procurement costs in Eqs. (7) and (10) when $c(x) \equiv 0.5x^2$.

APPENDIX

Proof of Propositions 2 and 3 . We proceed by considering that an amount $Y \geq 0$ is set aside for small firms. In case of Proposition 2, this amount is restricted to be zero. We make it explicit just at the end of the proof.

Let us suppose that winners in an auction for lots are not active in the competitive market and that small firms only bid for small lots (we will check later that this is indeed optimal in equilibrium). Suppose that we have $m_k \leq n_k - 1$ large suppliers already committed to produce one (large) lot each on the list $\mathbf{z} = (z_1, ..., z_{m_k})$ with $\sum_{i=1}^{m_k} z_i = Z$, and who are not interested in increasing their production (to be checked later). Similarly, suppose that we have $m_1 \leq n_1 - 1$ small suppliers already committed to produce one (small) lot each on the list $\mathbf{y} = (y_1, ..., y_{m_1})$ with $\sum_{i=1}^{m_1} y_i = Y$.[18] In the aftermarket, large suppliers sell quantity Q and small suppliers quantity q each, that satisfy the equilibrium conditions (2), and therefore $Q = kq$, and

$$(n_k - m_k)Q + (n_1 - m_1)\, q = X - Z - Y. \tag{11}$$

This leads to $q = \frac{X-Z-Y}{k(n_k-m_k)+n_1-m_1}$ and profits of suppliers in the market $\pi_1(q) \equiv qc'(q) - c(q)$ and $\pi_k(kq) \equiv kqc'(q) - kc(q)$. Note that $\pi_k(kq) = k\pi_1(q)$ and $\pi_1'(q) = qc''(q)$. For a lot z_i, large suppliers bid $T_{z_i} = kc(\frac{z_i}{k}) + k\pi_1(q)$, as in equilibrium their profits must be equal in the auction and in the market, just as in Bru et al. (2023). Similarly, $T_{y_i} = c(y_i) + \pi_1(q)$. Thus, procurement costs are

$$PC(\mathbf{z}, \mathbf{y}) = \sum_{i=1}^{m_k} kc\left(\frac{z_i}{k}\right) + \sum_{j=1}^{m_1} c(y_j) + (km_k + m_1)\, \pi_1(q) + c'(q)\,(X - Z - Y). \tag{12}$$

The only terms in (12) that are affected by the size distribution of the lots are $\sum_{i=1}^{m_k} kc\left(\frac{z_i}{k}\right)$ and $\sum_{j=1}^{m_1} c(y_j)$. By the convexity of $c(.)$, these terms are minimized when setting symmetric

[18] We will check at the end of this proof that it is indeed never optimal to set as many lots as there are suppliers of either type.

lots. Thus, we have that procurement costs are

$$PC(Z,Y,m_k,m_1) = m_k k c\left(\frac{Z}{km_k}\right) + m_1 c\left(\frac{Y}{m_1}\right) + (km_k + m_1)\,\pi_1(q) + c'(q)(X - Z - Y). \tag{13}$$

Given m_k symmetric lots, the optimal quantity Z auctioned off satisfies the first-order condition[19]

$$\frac{\partial PC(Z,Y,m_k,m_1)}{\partial Z} = c'\left(\frac{Z}{km_k}\right) - c'(q) - qc''(q)\frac{kn_k + n_1}{k(n_k - m_k) + n_1 - m_1} = 0. \tag{14}$$

Similarly,

$$\frac{\partial PC(Z,Y,m_k,m_1)}{\partial Y} = c'\left(\frac{Y}{m_1}\right) - c'(q) - qc''(q)\frac{kn_k + n_1}{k(n_k - m_k) + n_1 - m_1} = 0. \tag{15}$$

It is straightforward to check that it is optimal to have the market functioning: There is an active market, since if $q = 0$ then $Z + Y = X$, and thus either $\frac{\partial PC}{\partial Z} = c'\left(\frac{Z}{km_k}\right) > 0$ or $\frac{\partial PC}{\partial Y} = c'\left(\frac{Y}{m_1}\right) > 0$, contradicting optimality. Moreover, not only is it optimal to hold both auctions, $Z^* > 0$ and $Y^* > 0$, but indeed winners in an auction will not participate in the competitive market, since it is optimal to auction off quantities Z^* and Y^* such that winners produce more than (similar) participants in the market, $\frac{Z^*}{km_k} > q$, and $\frac{Y^*}{m_1} > q$ and therefore their marginal costs are too high. To see this, observe that by the second-order condition, both left-hand sides are increasing, in Z and Y respectively. Next, note that at $\frac{Z^*}{km_k} = q$, the first two terms in (14) cancel, and thus, by $q > 0$ and the convexity of $c(.)$, $\frac{\partial PC}{\partial Z} < 0$. For a smaller Z the same two terms add a negative amount, so the derivative is even lower. The same argument works for the small firms. Finally, note that, according to (14) and (15), the optimal size of lots are such that production per plant of firms (large or small) that receive a lot must be the same; that is, $q_L^* = \frac{Z^*}{km_k} = \frac{Y^*}{m_1}$.

Define $n = kn_k + n_1$ the total number of plants in the industry and $m = km_k + m_1$ the total number of plants that produce lots, and $q^* = \frac{X - mq_L^*}{n - m}$ the production per plant of firms that (only) produce for the market. We can write (14) and (15) as a unique first-order condition

$$c'(q_L^*) - c'(q^*) - q^*c''(q^*)\frac{n}{n - m} = 0. \tag{16}$$

[19] By our assumption on $c'''(.)$ the second-order condition for optimality is satisfied.

and procurement costs as

$$PC\left(m,q^{*},q_{L}^{*}\right)=mc\left(q_{L}^{*}\right)+m\pi_{1}\left(q^{*}\right)+(n-m)c'\left(q^{*}\right)q^{*}. \tag{17}$$

To see the optimal number of lots, we differentiate total costs with respect to m, to obtain

$$\begin{aligned}\frac{\partial PC\left(m,q^{*},q_{L}^{*}\right)}{\partial m}&=c\left(q_{L}^{*}\right)+\pi_{1}\left(q^{*}\right)-c'\left(q^{*}\right)q^{*}\\&-\left[nc''(q^{*})q^{*}+(n-m)c'(q^{*})\right]\frac{q_{L}^{*}-q^{*}}{n-m}.\end{aligned} \tag{18}$$

Substituting in from (16) we get

$$\frac{\partial PC\left(m,q^{*},q_{L}^{*}\right)}{\partial m}=c\left(q_{L}^{*}\right)-c(q^{*})-c'\left(q_{L}^{*}\right)\left(q_{L}^{*}-q^{*}\right), \tag{19}$$

which can be written as

$$\int_{q^{*}}^{q_{L}^{*}}\left[c'(s)-c'\left(q_{L}^{*}\right)\right]ds,$$

what is clearly negative. Therefore, for $0\leq m_{k}\leq n_{k}-1$, $m_{k}^{*}=n_{k}-1$; and for $0\leq m_{1}\leq n_{1}-1$, $m_{1}^{*}=n_{1}-1$.

Finally, let us show that – given the above – a small firm would not wish to win a large lot. Note that the net effect of giving up on participating in the market in exchange for producing a large lot and being paid the equilibrium bid of the large firms is[20]

$$kc\left(\frac{Z}{k(n_{k}-1)}\right)+k\pi_{1}(q)-c\left(\frac{Z}{n_{k}-1}\right)-\pi_{1}(q).$$

Since, as we have seen, $\frac{Z}{k(n_{k}-1)}>q$, the above expression is less than

$$\begin{aligned}&kc\left(\frac{Z}{k(n_{k}-1)}\right)+(k-1)\pi_{1}\left(\frac{Z}{k(n_{k}-1)}\right)-c\left(\frac{Z}{n_{k}-1}\right)\\&=kc\left(\frac{Z}{k(n_{k}-1)}\right)-c\left(\frac{Z}{n_{k}-1}\right)+(k-1)\left(\frac{Z}{k(n_{k}-1)}c'\left(\frac{Z}{k(n_{k}-1)}\right)-c\left(\frac{Z}{k(n_{k}-1)}\right)\right)\\&=c\left(\frac{Z}{k(n_{k}-1)}\right)-c\left(\frac{Z}{n_{k}-1}\right)+\left(\frac{Z}{n_{k}-1}-\frac{Z}{k(n_{k}-1)}\right)c'\left(\frac{Z}{k(n_{k}-1)}\right)\\&=-\int_{\frac{Z}{k(n_{k}-1)}}^{\frac{Z}{n_{k}-1}}\left[c'(x)-c'\left(\frac{Z}{k(n_{k}-1)}\right)\right]dx<0.\end{aligned}$$

[20] Participating in the market or winning a small lot give the same equilibrium profit $\pi_1(q)$ to small firms.

Thus, the deviation is not profitable.

Up to now we have supposed that it makes no sense to set n_k large lots. To check that, suppose otherwise. Suppose also that the optimal small-lot auction has m lots and consequently, the market will have $n_1 - m$ small firms in it. Continuing with the assumption that the outcome is competitive even when there is a single firm in it (basically, allowing the buyer to set a unit price he is willing to pay and allowing the supplier to choose the quantity it will supply at that price), we obtain that $q = \frac{X-Z-Y}{n_1-m}$, $B_Z = c\left(\frac{Z}{n_k}\right) + \pi_1(q)$ and $B_Y = c\left(\frac{Y}{m}\right) + \pi_1(q)$. Thus, total cost becomes

$$\begin{aligned} PC &= n_k\left(c\left(\frac{Z}{n_k}\right) + \pi_1(q)\right) + (n_1-1)\left(c\left(\frac{Y}{m}\right) + \pi_1(q)\right) + c(q) + \pi_1(q) \\ &= n_k c\left(\frac{Z}{n_k}\right) + (n_1-1)\, c\left(\frac{Y}{m}\right) + c(q) + (n_k+n_1)\,\pi_1(q). \end{aligned}$$

We obtain the optimality conditions

$$c'\left(\frac{Z}{n_k}\right) - c'(q) - (n_k+n_1)\, qc''(q) = 0$$

and

$$c'\left(\frac{Y}{m}\right) - c'(q) - (n_k+n_1)\, qc''(q) = 0.$$

From here we see that the lot sizes are equal in the two auctions and consequently, it is as if we had a single auction, what is clearly not an improvement. Finally, suppose $m_1 = n_1$. Then, as there is no competition for the last lot, small firms ask for the moon.

Substituting $m_k^* = n_k - 1$ and $m_1^* = n_1 - 1$ into (13), (14) and (15), we obtain (8) and (9); thus, proving Proposition 3.

Similarly, by letting $Y = 0$ (thus, $m_1 = 0$) and substituting $m_k^* = n_k - 1$ into (13) and (14), we obtain (5) and (6). Moreover, the non-profitability of winning a large lot is independent of Y being positive or not. So, Proposition 2 is also proved. ■

Proof of Corollary 6. As we have seen, the profit of each small firm is $qc'(q) - c(q)$ in each mechanism. As $qc'(q) - c(q)$ is increasing in q the ranking of profits is the same as the ranking of a small firm's production in the respective mechanisms. To see that $q_{EU}^* > q_{US}^*$ note that by (5) and (8)

$$\frac{kn_k + n_1}{k+n_1} c''\left(q_{EU}^*\right) q_{EU}^* + c'\left(q_{EU}^*\right) = c'\left(\frac{Z_{EU}^*}{k(n_k-1)}\right)$$

and

$$\frac{kn_k + n_1}{k+1} c''(q_{US}^*)\, q_{US}^* + c'(q_{US}^*) = c'\left(\frac{Z_{US}^*}{k(n_k - 1)}\right).$$

Suppose for contradiction that $q_{EU}^* \leq q_{US}^*$. Then, since by our assumption on $c'''(.)$ – c.f. footnote 6 – $c''(q)\, q$ is increasing, the left-hand side in the US would be higher than in the EU, implying $Z_{US}^* > Z_{EU}^*$. But then – given that the lots of the set-aside auction exceed q_{US}^*, the overall purchase in the US would be higher than in the EU, contradicting the fact that it is X in both cases.

The comparison with q_M follows from Corollary 3, as higher lots leave less quantity to be procured in the market. ■

Proof of Corollary 7. If we wish the small firms in the US market to produce q_{EU}^* then the large firm in the market will produce kq_{EU}^*, and the amount procured by the two auctions are Z_{USh} and Y_{USh}. Then $X - Z_{USh} - Y_{USh} = (k+1)q_{EU}^*$. Assume that, for efficiency, we set $\frac{Z_{USh}}{n_k - 1} = k\frac{Y_{USh}}{n_1 - 1}$. The resulting production cost can be written as

$$C_{USh} = (n_k - 1)kc\left(\frac{Z_{USh}}{k\,(n_k - 1)}\right) + (n_1 - 1)c\left(\frac{Y_{USh}}{n_1 - 1}\right) + (k+1)c(q_{EU}^*) \tag{20}$$

$$= (k\,(n_k - 1) + n_1 - 1)\, c\left(\frac{Y_{USh}}{n_1 - 1}\right) + (k+1)c(q_{EU}^*). \tag{21}$$

The production costs in the European system are

$$C_{EU} = (n_k - 1)kc\left(\frac{Z_{EU}}{k(n_k - 1)}\right) + (k + n_1)c(q_{EU}^*).$$

Since $X - Z_{EU} = (k + n_1)q_{EU}^*$ and

$$X - Z_{USh} - Y_{USh} = X - (k(n_k - 1) + n_1 - 1)\,\frac{Y_{USh}}{n_1 - 1} = (k+1)q_{EU}^*,$$

we obtain

$$Z_{EU} = (1 - n_1)q_{EU}^* + (k(n_k - 1) + n_1 - 1)\,\frac{Y_{USh}}{n_1 - 1}$$

and thus,

$$\frac{Z_{EU}}{k(n_k - 1)} = \frac{Y_{USh}}{n_1 - 1} + \frac{\left(\frac{Y_{USh}}{n_1 - 1} - q_{EU}^*\right)(n_1 - 1)}{k(n_k - 1)}. \tag{22}$$

We can now rewrite C_{EU} as

$$C_{EU} = (n_k-1)kc\left(\frac{Y_{USh}}{n_1-1}+\frac{\left(\frac{Y_{USh}}{n_1-1}-q^*_{EU}\right)(n_1-1)}{k(n_k-1)}\right)+(n_1-1)\,c(q^*_{EU})+(k+1)c(q^*_{EU}). \tag{23}$$

Comparing (23) and (20) it is immediate that if and only if $\frac{Y_{USh}}{n_1-1} > q^*_{EU}$ the production costs are higher in the European system. To see that the inequality holds, suppose for contradiction that $\frac{Y_{USh}}{n_1-1} \leq q^*_{EU}$. Then by (22)

$$\frac{Z_{EU}}{k(n_k-1)} \leq \frac{Y_{USh}}{n_1-1} \leq q^*_{EU}.$$

However, note that by (5)

$$\frac{Z_{EU}}{k(n_k-1)} > q^*_{EU},$$

what is a contradiction.

Finally, since the suppliers' profits are equal across the two systems, the (restricted) US system must lead to lower procurement costs, as they are the sum of production costs plus supplier profits. ■

Proof of Corollary 8. The quantity produced in the single market by all the small firms combined is $n_1 q_M = \frac{Xn_1}{kn_k+n_1}$, while in the US mechanism it is

$$q^*_{US}+Y^*_{US} = \frac{X\,(k+1)}{(kn_k+n_1)^2}+X(n_1-1)\frac{k(n_k+1)+n_1+1}{(kn_k+n_1)^2}.$$

Subtracting the former from the latter we obtain $Xk\frac{n_1-n_k}{(n_1+kn_k)^2}$. In the European system all the small firms sell in the aftermarket: $n_1 q^*_{EU} = Xn_1\frac{k+n_1}{(kn_k+n_1)^2}$.

$$q^*_{US}+Y^*_{US}-n_1 q^*_{EU} = \frac{Xkn_k\,(n_1-1)}{(kn_k+n_1)^2} > 0.$$

Similarly,

$$n_1 q_M - n_1 q^*_{EU} = \frac{Xkn_1\,(n_k-1)}{(kn_k+n_1)^2} > 0.$$

■

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